Algebraic Topology WS26
Exercise set 4.

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All homology groups are taken with integer coefficients.

Problem 1. Use van Kampen's theorem to prove that S^n is simply connected for $n \geq 2$. Where does the same argument fail for $n = 1$?

Problem 2. Compute the simplicial homology groups of S^2 using the triangulation with four triangles given by the boundary of a tetrahedron.

Problem 3. [Hatcher, Ch. 2.1, Ex. 1] What familiar space is the quotient of a 2-simplex $[v_0, v_1, v_2]$ obtained by identifying the edges $[v_0, v_1]$ and $[v_1, v_2]$, preserving the ordering of vertices? Compute its homology.

Problem 4. Let $f, f' : C_* \to D_*$ and $g, g' : D_* \to E_*$ be maps of chain complexes. Show that if $f \simeq f'$ and $g \simeq g'$ by chain homotopies, then $g \circ f \simeq g' \circ f'$ by a chain homotopy.

Problem 5. Let C_* be the chain complex with $C_1 = C_0 = \mathbb{Z}$, $\partial_1 : C_1 \to C_0$ given by $\partial_1(x) = 2x$, and $C_i = 0$ for all other i. Let D_* be the chain complex with $D_0 = \mathbb{Z}/2\mathbb{Z}$ and $D_i = 0$ for $i \neq 0$. Show that C_* and D_* have isomorphic homology groups in every degree, but are not chain-homotopy equivalent.

Due date: Monday, 16.11.2026