

Algebraic Topology WS26

Exercise set 3.

Lecturer: Anton Mellit
Exercise sessions: Livia Campo

Problem 1. [Hatcher, Ch. 1.3, Ex. 2] Show that if $p_i : \tilde{X}_i \rightarrow X_i$ are coverings for $i = 1, 2$, then $p_1 \times p_2 : \tilde{X}_1 \times \tilde{X}_2 \rightarrow X_1 \times X_2$ is a covering. Suppose X_1 and X_2 are path-connected and choose $x_i \in X_i$. Describe the monodromy action of $\pi_1(X_1 \times X_2, (x_1, x_2))$ on the fibre over (x_1, x_2) in terms of the actions on $p_i^{-1}(x_i)$.

Problem 2. [Hatcher, Ch. 1.3, Ex. 9] Show that if a path-connected, locally path-connected space X has finite fundamental group, then every continuous map $f : X \rightarrow S^1$ is null-homotopic. *Hint.* Use the covering $\mathbb{R} \rightarrow S^1$.

Problem 3. [Hatcher, Ch. 1.3, Ex. 8] Let X and Y be path-connected, locally path-connected and semilocally simply connected spaces. Suppose that X and Y are homotopy equivalent. Prove that their universal covering spaces $\tilde{X}$ and $\tilde{Y}$ are homotopy equivalent.

Problem 4. [Hatcher, Ch. 1.3, Ex. 10] Find all connected two-sheeted and three-sheeted covering spaces of $S^1 \vee S^1$, up to isomorphism of covering spaces without basepoints.

Problem 5. Let X be S^2 with three points removed. Construct a connected covering $p : E \rightarrow X$ with more than one sheet and with trivial automorphism group $\text{Aut}(p)$. Here $\text{Aut}(p)$ consists of the homeomorphisms $g : E \rightarrow E$ satisfying $p \circ g = p$.

Due date: Monday, 09.11.2026